

Model PERT Project Schedules with the BETA Distribution Using EXCEL

PERT uses estimates of task times to compute statistical variations in project schedules. Since schedules have defined starting and ending points and as it is more likely that completion occurs later and later, the asymmetric Beta distribution models schedules very well. It was used to estimate schedules since the creation of PERT (circa 1958).

Project management texts describe PERT as a useful scheduling method and connect it to the Beta distribution, but leave out the details of how to compute it and use it. Supplemental or add on software is not required - EXCEL may be used to compute the Beta probability density from the normal PERT estimates.

Beta Function

The Beta *function* is needed to compute the Beta *probability density*. The Beta function is not directly contained in EXCEL (discussions here use EXCEL version 2000); rather, it is computed from the GAMMA function which is contained in EXCEL in the form *GAMMALN*. The latter is the logarithmic form of the GAMMA function.

Generally, with $\Gamma(a) = EXP[GAMMALN(a)]$ and $\Gamma(b) = EXP[GAMMALN(b)]$ and

$$\Gamma(a + b) = EXP[GAMMALN(a + b)],$$

the Beta function is computed as:

$$Beta(a, b) = \frac{\Gamma(a) \times \Gamma(b)}{\Gamma(a + b)} \text{ for values } a, b.$$

PERT Derived Parameters

The PERT independent estimates are labeled *Min*, *Max*, and *Mode*. Recall that these are respectively, the shortest, longest, and most likely estimates for the durations to complete a task/project.

The derived parameters are:

$$Mean = \frac{(Max + 4 \times Mode + Min)}{6},$$

$$StdDev = \frac{(Max - Min)}{6}, \quad \text{and}$$

$$Skew = \frac{2(ShapeB - ShapeA)}{(ShapeA + ShapeB + 2)} \times \left(\frac{(ShapeA + ShapeB + 1)}{(ShapeA \times ShapeB)} \right)^{0.5}.$$

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The two Shape parameters are obtained as:

$$ShapeA = \left[\frac{(Mean - Min)}{(Max - Min)} \times \frac{(Mean - Min) \times (Max - Mean)}{StdDev^2} - 1 \right] \quad \text{and}$$

$$ShapeB = \frac{(Max - Mean)}{(Mean - Min)} \times ShapeA.$$

Beta Probability Density

One can then compute the Beta Probability Density from the PERT estimates. Using the above parameters, the *Beta Probability Density* function becomes:

$$BetaDensity(x) = \frac{(x - Min)^{(ShapeA-1)} \times (Max - x)^{(ShapeB-1)}}{Beta(ShapeA, ShapeB) \times (Max - Min)^{(ShapeA+ShapeB-1)}}.$$

Cumulative Beta

EXCEL will compute the cumulative Beta distribution directly as:

BETADIST(x, ShapeA, ShapeB, Min, Max).

EXCEL also contains the function *BETAINV*, which is the inverse of the cumulative distribution. This is useful to find specific time intervals at which the probability reaches a given value – the 50%, 75%, 90%, and 95% levels are usually of interest. Thus, to find the time that the task will be completed with a confidence of 95%, enter:

BETAINV(0.95, ShapeA, ShapeB, Min, Max).

Example and Interpretation

An estimate of a task schedule provided by a task manager is as follows:

<i>Min</i>	10.0	Weeks
<i>Max</i>	25.0	Weeks
<i>Mode</i>	12.0	Weeks

The PERT parameters derived from these estimates are:

<i>Mean</i>	13.8	Weeks
<i>Std. Dev.</i>	2.50	Weeks
<i>Skew</i>	0.75	
<i>Shape A</i>	1.4947	
<i>Shape B</i>	4.3542	

The Beta distributed schedule with these parameters is generated by calculating the probability density at assumed time intervals and is illustrated as Figure 1.

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While the task manager estimated the task will most likely be completed in 12 weeks (the *Mode* value), the *Mean* value is 13.8 weeks. The *Mean* is skewed to the right (positive skew) of the *Mode*, or further in time by 1.8 weeks.

From the cumulative distribution shown as Figure 2, there is a 95% chance that the task will finish in 18.6 weeks. The *Median* value is 13.4 weeks. Specific percentile values are obtained using the function *BETAINV* and representative Percentiles are shown in Table 1. Note that using *BETAINV* does not require computation of the Beta probability density.

Table 1

	Percentile	Weeks
	25.0%	11.8
Median	50.0%	13.4
	75.0%	15.4
	90.0%	17.4
	95.0%	18.6

Quick Estimation

By making the assumption that the Beta probability density is approximated by a Normal probability density, there is a 95.5% chance that the range of duration in schedule is contained within the values: $Mean \pm 2xStd.Dev.$

For this example this range is 13.8 weeks $\pm 2x2.5$ weeks or between 8.8 and 18.8 weeks. Normally, the lower limit is of no concern and for this example is computed to be lower than the task manager's estimate. The upper limit of 18.8 weeks agrees very well with the 18.6 weeks computed at the 95% level using the exact cumulative Beta function. In Figure 1 note that while the distribution appears to be rather asymmetric, the quick estimate for 95% completion is nonetheless quite good.

Comments

Using derived PERT parameters for *Mean* and *Std.Dev.* for rapid estimation, there is a 95% confidence that the schedule will be between $Mean \pm 2xStd.Dev.$ This is certainly acceptable if the shape is as indicated in Figure 1. This is the typical shape since most scheduled tasks/projects complete later than predicted – i.e., there are more dates later than the most likely date and fewer dates earlier than the most likely date. The PERT weighted average shifts to a position later than the most likely date (the *Mode*).

An exact computation would use $BETAINV(0.95, ShapeA, ShapeB, Min, Max)$ to compute the cumulative probability and requires only the computation of the additional parameters *ShapeA* and *ShapeB* which then allows computation at any percentile.

An issue is: is this too much estimation? The Beta distribution might be more useful when applied to the entire project schedule rather than specific tasks. The project schedule should include only those tasks on the critical path. The project duration is the sum of the individual task PERT mean values and the project standard deviation is the root-sum-of-squares of the individual PERT standard deviations.

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See "Beta Modeled PERT Schedule Example" EXCEL workbook on WEB Site.

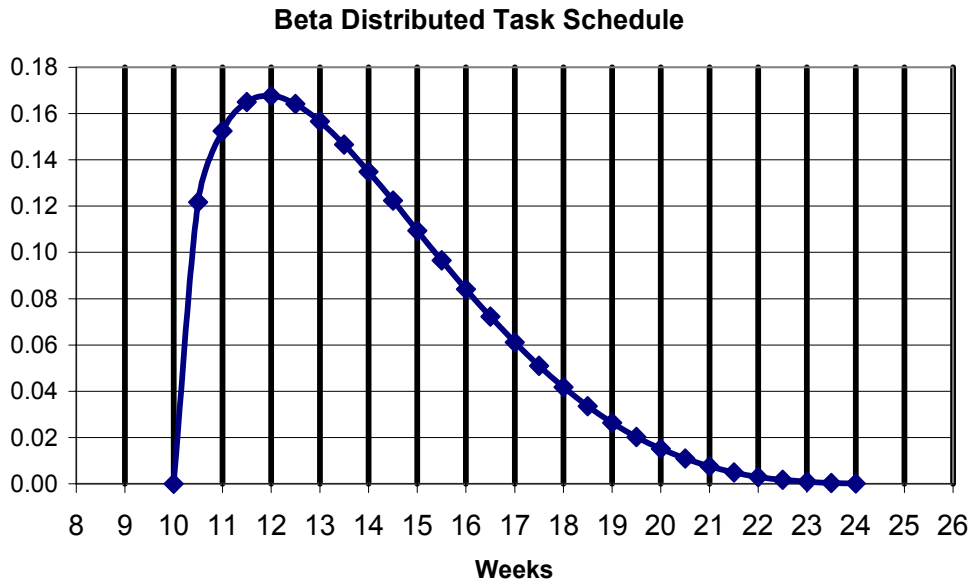


Figure 1 Schedule Modeled as Beta Distribution

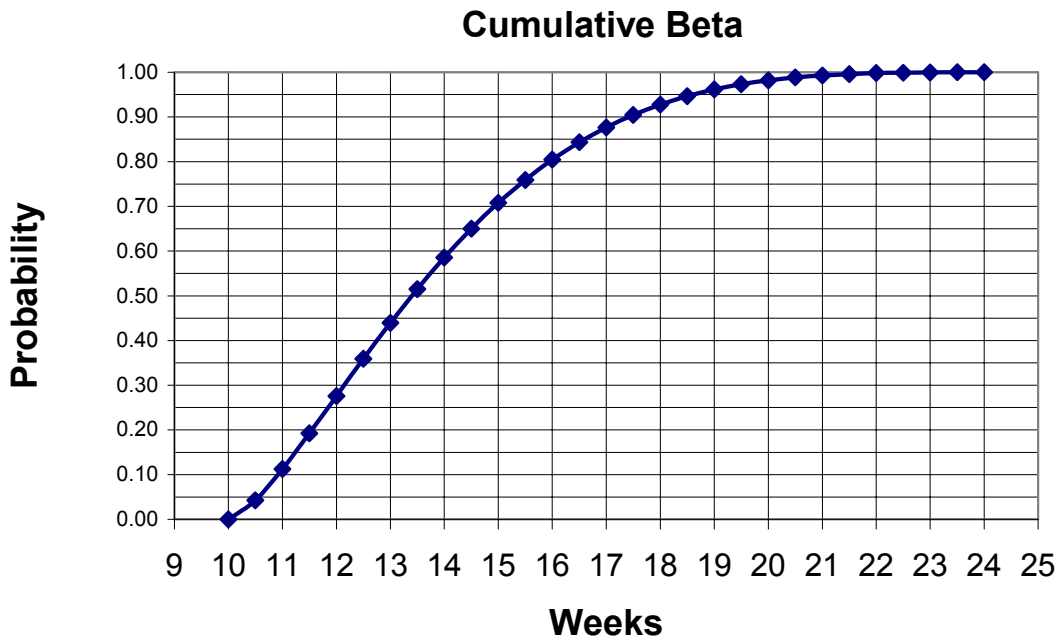


Figure 2 Probability to Complete Based on Beta Model

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